

A-Posteriori Error Estimation for High Order Accurate Godunov Finite Volume Methods

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A frequent objective in numerically solving partial differential equations is the subsequent calculation of certain derived quantities of particular interest, e.g., aerodynamic lift and drag forces, stress intensity factors, mean temperatures, etc. Consequently, there is considerable interest in constructing *a posteriori* error estimates for such derived quantities (mathematically described as functionals). A further motivation for the construction of *a posteriori* error estimates comes from the subsequent derivation of local cell refinement indicators for use in mesh or solution adaptivity. The ability to compute the error in a given numerical computation and improve upon it by local mesh or solution adaptivity are two essential ingredients in the construction of efficient and reliable computational methods.

This work [1] considers *a posteriori* error estimates and mesh refinement indicators for high order Godunov finite volume methods that utilize the two solution representations inherent in the method, viz. as piecewise constants u_0 and cellwise p -th order reconstructed functions $R_p^0 u_0$. Using standard duality arguments (see [2] and [3]) an *exact* error representation formula is derived for user specified functionals that is tailored to the class of high order Godunov finite volume methods with data reconstruction, $R_p^0 u_0$. Computable error estimates are then devised that exploit the structure of Godunov finite volume methods. The present theory applies directly to a wide range of finite volume methods in current use including MUSCL, TVD, UNO, and ENO methods. The present techniques are particularly effective for hyperbolic problems where the long range propagation of solution errors can dramatically affect the mesh adaptation process. Additional issues such space-time error estimation, the treatment of nonlinearity, and the post-processing of dual problem data are considered as well. Numerical results for conservation laws are presented to validate various aspects of the analysis.

- [1] T. Barth and M. Larson, "A *Posteriori* Error Estimates for Higher Order Godunov Finite Volume Methods on Unstructured Meshes", Finite Volumes for Complex Applications III, Hermes Pub. Ltd., June 2002
- [2] K. Eriksson and D. Estep and P. Hansbo and C. Johnson, "Introduction to Numerical Methods for Differential Equations," *Acta Numerica*, 1995
- [3] R. Becker and R. Rannacher, "An Optimal Control Approach to A-Posteriori Error Estimation in Finite Element Methods," *Acta Numerica*, 2001