Multi-scale representations as a tool for adaptive numerical schemes for systems of conservation laws.

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State of the art numerical simulations for systems of conservation laws succeed in obtaining highly accurate numerical approximations in smooth regions of the flow regime, together with sharp, oscillation free, profiles at shocks and material interfaces.

There is a now a variety of so-called High Resolution Shock Capturing (HRSC henceforth) schemes to be used in numerical simulations involving Hyperbolic systems of conservation laws in various scenarios. In a HRSC scheme of order larger than or equal three, the heart of the scheme is the computation of the numerical flux function. This is also the most time consuming process in the overall computation. The numerical flux function essentially characterizes the choice of scheme, and it has been shown over the years that for a given problem, the particular choice of the numerical flux function chosen may have a definite impact in the overall quality of the computed solution.

The robustness of the different HRSC schemes proposed over the years by different authors has been extensively tested in numerous works, and it is generally agreed that, while a simple numerical flux function might be sufficient in may situations, to avoid pathological behavior of numerical nature, one has to resort to more sophisticated schemes, with considerably more expensive (from a computational point of view) numerical flux functions. However, it is also agreed that in nearly all cases, oscillations and other undesirable behavior only occur because unsophisticated flux functions fail near discontinuities, or when these are ready to be formed. This observation was the main motivation behind the work of Ami Harten. ENO (Essentially Non Oscillatory) schemes are now commonly used in numerical simulations involving systems of conservation laws. The technology introduced by Ami Harten and his collaborators has proven to be very fruitful, and it has lead to many robust numerical schemes. However, high order ENO-type numerical flux functions come associated with an important numerical expense.

In this paper we will review Ami Hartens seminal work on the use of multi-scale decompositions to reduce the work associated to HRSC numerical simulations. In particular, we will concentrate on the contributions of several researchers (including the authors) that have followed the path laid out by Harten and have obtained what we think are state of the art, competitive, multi-level schemes.