A Hierarchical Adaptive Grid Code for Non-linear Elastodynamics

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Since the equations governing the dynamics of hyperelastic materials are a set of nonlinear hyperbolic conservation laws, the most appropriate way to solve these equations numerically is to use a characteristic based conservative scheme. However, it is generally believed that the complexity of associated Riemann problem makes the computational cost of such schemes prohibitive. Here we show that this difficulty is more apparent than real and that it is possible to devise a method that is almost as fast as the conventional finite element schemes. This has several advantages: it is much more robust; it has excellent shock capturing properties; a characteristic based algorithm is essential for an adaptive scheme in order to avoid the generation of errors at refinement boundaries. As one would expect, the scheme is excellent at propagating the waves that occur in dynamic problems and, when combined with a multigrid, it is also very effective at finding equilibrium solutions.

Using such a scheme, it is possible to study a number of interesting properties of such systems, such as the existence, or otherwise, of non-evolutionary shocks and the appearance of stress singularities at stress-free boundaries. The latter are particularly important because they cause considerable difficulties for conventional finite element methods.

It is well known that, in linear elasticity, a concave corner in an elastic body with a stress free boundary leads to a singularity, but that such a corner cannot appear if it is not present in the initial configuration. In the nonlinear case a body with an initially smooth boundary can develop corners if the deformation is sufficiently large, but, since the location of this singularity is not necessarily known a priori, a dynamically adaptive grid is required to ensure high resolution in the neighbourhood of the singularity. However, the mere use of an adaptive grid is not enough unless something is also done about the singularity.

An effective way of dealing with this is to model a stress free boundary by immersing the elastic body in a very weak material with a smooth transition in properties between the two materials. This removes the singularity, but its nature can be studied by varying the thickness of the transition region. This technique has the additional advantage that it provides a simple way of representing complex body shapes on a cartesian grid. For this an adaptive grid is essential, since an accurate representation of the body would be prohibitively expensive with a uniform grid.

These ideas will be illustrated by examples in which the material law models polyurethane foams and the configurations are relevant to various industrial applications.